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Editorial

EDITORIAL

Improving pulse oximetry accuracy in dark-skinned patients: technical aspects and current regulations

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Summary

Recent concerns regarding the clinical accuracy of pulse oximetry in dark-skinned patients, specifically in detecting occult hypoxaemia, have motivated research on this topic and recently reported in this journal. We provide an overview of the technical aspects of the issue, the sources of inaccuracy, and the current regulations and limitations. These insights offer perspectives on how pulse oximetry can be improved to address these potential limitations.

Keywords: clinical accuracy; dark skin; hypoxaemia; melanin; pulse oximetry; regulations

Progress in portable photoplethysmography devices for continuous monitoring of physiological variables has created a new opportunity for a proactive approach to healthcare.[1] Because of its popularity and widespread acceptance in many hospital and home settings, pulse oximetry has established itself as the gold standard in primary care by providing relevant data to identify health issues at an early stage. In particular, pulse oximetry has been widely used in patients with COVID-19 for staging disease severity and as an indication for and monitoring of supplemental oxygen therapy.[2] Because of the recent availability of 'over-the-counter' pulse oximeters, the accuracy of these devices has received a great deal of attention, driven by the need to identify occult hypoxaemia (arterial oxygen saturation [SaO_2] <88% despite normal peripheral blood oxygen saturation [SpO_2] >92%) quickly and accurately during the progression of COVID-19.[3,4]

A key element in clinical decision-making is to have a deeper understanding of the errors that can affect pulse oximetry accuracy. Although several factors can influence the accuracy of pulse oximetry, the impact of skin pigmentation has recently received the most attention, with large-scale studies observing inaccuracies and overestimates in individuals with higher skin melanin concentration.[5] Recent research has found that pulse oximeters fail to detect hypoxaemia more frequently in subjects with darkly pigmented skin.[6,7] An observational study conducted in 26,603 patients with different skin melanin concentrations admitted to the ICU of two US academic medical centres (Mayo Clinic in Minnesota, FL, USA and in Phoenix, AZ, USA) with paired simultaneous measurements of SpO_2 and SaO_2 found that the probability of occult hypoxaemia increased with lower SpO_2 values and was higher in patients with dark pigmentation.[8] Another study conducted in 324 centres in the USA in patients with respiratory failure found that the prevalence of occult hypoxaemia was higher in darkly pigmented patients. In particular, the rate for 186 patients with light pigmentation was 10.2% compared with 21.5% for 51 patients with dark pigmentation. Additionally, patients with darker pigmentation were reported to have a three times higher (statistically significant) risk of occult hypoxaemia.[4] Another recent study of 1061 children also found that 21.1% of children with darker pigmentation had occult hypoxaemia.[9] In SpO_2–SaO_2 pairs,

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the odds of the SpO_2 reading failing to detect arterial hypoxaemia were more than doubled if the pair came from these children.

The presence of occult hypoxaemia is a risk factor associated with increased mortality. It can delay diagnosis and subsequent initiation of treatment. When the patient and caregivers are given a false sense of well-being, there is a corresponding risk of suffering from complications that are not quickly detected. Such bias is of great concern, as inaccuracies as low as 2% are of special concern in respiratory rehabilitation, sleep apnoea studies, or athletes performing physical exertion. They can lead to serious issues for the patient, requiring external oxygen supply or even hospitalisation. Because of this growing evidence, several health organisations and health professionals have indicated the need to introduce improvements in the design of pulse oximeters for use in patients with dark pigmentation and improved understanding of the effects of skin pigmentation on oximetry accuracy.[10–12]

In a recent issue of the *British Journal of Anaesthesia*, Rea and Bierman[13] present the provocative hypothesis that the oxygen saturation inaccuracy seen in individuals with darker skin pigments is caused by poor-quality light-emitting diodes (LEDs). Specifically, the LEDs used in many commercial pulse oximeters do not provide monochromatic light at the specified wavelength. Whilst these LEDs provide light at the required wavelength, their light is contaminated by a polychromatic spillover from nearby wavelengths. In the words of the researchers, ‘ …spectral absorption of melanin interacts with the polychromatic LED light sources, resulting in a positive drift from the calibration curve at higher melanin concentrations’. From studies, such as this, it can be hoped that a technical solution may be developed for this vexing problem.

Technical aspects of the problem

Understanding the principles underlying pulse oximetry is crucial for overcoming its limitations. Pulse oximeters estimate oxygen saturation SpO_2 by analysing optical signals between blood and surrounding tissue, eliminating the need for invasive procedures, such as arterial puncture. Traditional pulse oximeters consist of two main components: a LED emitting light at two or more wavelengths and a photodetector. By illuminating a tissue bed, such as the finger, with different wavelengths of light and measuring the amount of absorbed light, two or more photoplethysmography pulse-wave optical signals are generated. The cardiac modulation of these signals creates a pulse wave, where the received light decreases and then increases with each heartbeat as the tissue blood volume changes during systole and diastole. Photoplethysmography signals are obtained simultaneously for different wavelengths, with one wavelength more influenced by oxygenated blood and the other more influenced by deoxygenated blood. These differences allow for continuous measurements of cardiovascular parameters, such as HR, peripheral blood oxygen saturation SpO_2, and peripheral blood flow (perfusion index).

Determination of tissue oxygen saturation SpO_2 relies on optical quantification of oxyhaemoglobin (HbO_2) concentration. HbO_2 absorbs visible and infrared light differently compared with deoxyhaemoglobin (HHb), which appears darker and has a bluish (cyanotic) hue under white light illumination. The transmission of light through the blood flow is influenced by the absorption coefficients at the two typical wavelengths used, namely red and infrared. By isolating the colour of the modulating signal component, the proportion of arterial blood containing either red HbO_2 or blue HHb can be determined.[14]

Spectrophotometric methods enable determination of oxygen saturation in an ideal medium through the measurement of optical absorption $A(\lambda)$ at two or more wavelengths. It is important to note that the specific algorithms used in pulse oximetry devices can vary amongst manufacturers and models. These algorithms often involve a combination of signal processing, data analysis, and statistical methods to estimate oxygen saturation accurately whilst considering baseline absorbance and tissue characteristics. A detailed description of the fundamental mathematics and algorithms used to convert measured light intensities into oxygen saturation values can be found in the Supplementary material.

To generate a reliable and robust photoplethysmography waveform from the backscattered light, it is crucial to consider the opacity of the skin and the optical absorption spectrum of the blood. Because of the high light absorption capacity of melanin, the transmitted light wavelength is reduced, making it unable to penetrate subcutaneous tissue.[15] Absorption of light by melanin differs at the two commonly used wavelengths. It absorbs more of the 660 nm light without significantly affecting the absorption of 900 nm light. Consequently, the presence of relatively high concentrations of melanin interacts with polychromatic LED light sources and causes a redshift in the center wavelength of the transmitted light. As the redshift magnitude increases with skin melanin concentration, overestimation errors are found in individuals with darker skin. Inclusion of additional wavelengths, two in the near-infrared and another two near the visible spectrum, improves determination of oxygen saturation in dark skin, as they are less sensitive to variations in the amount of oxygen bound in blood haemoglobin compared with classical devices. However, it is crucial to select the most appropriate wavelengths for each segment to minimise interference from skin melanin.

Studies have demonstrated that the design of devices with specific wavelengths, such as a first light generator alternately emitting signal A of a wavelength below 780 nm and a second generator emitting signal B above 800 nm, allows for reliable use of a noninvasive sensor on dark-skinned subjects.[16] Inclusion of other wavelengths does not imply that all four wavelengths need to be used simultaneously. In fact, using multiple wavelengths at the same time would have certain disadvantages, such as requiring a longer battery life to sustain power and potentially overheating the device. Instead, the device can selectively choose which wavelengths to use based on the individual’s skin colour. Analysis of photoplethysmography signals at different wavelengths has shown that not only is the accuracy of SpO_2 improved, but it is also possible to extract more information about skin pathologies at different tissue depths by being able to detect other substances, such as methaemoglobin and carboxyhaemoglobin.[17]

To date, most studies of pulse oximeters have been calibrated using individuals with low skin melanin concentration, whilst the population sample with dark pigmentation remains under-represented.[8,18] Furthermore, these studies have been conducted on a relatively small number of pulse oximeter models. The level of inaccuracy varies significantly amongst different devices, manufacturers, and probes. For example, data collected by the Open Oximetry Project (www.openoximetry.org) demonstrate the varying performances of

low-cost pulse oximeters. Most of these studies utilised subjective scales for population classification, such as the use of uncertain ancestry descriptors or ambiguous terms, such as race or ethnicity, which do not accurately capture skin tone and can present problems on multiple levels. Several studies use terms, such as Hispanic, Asian, American Indian, African American, and Native Hawaiian, or even White or Black,[6,7,19,20] for self-identification of subjects. These categorisations are broad and imprecise, failing to account for an accurate description of the full range of skin pigmentation and leading to inappropriate classifications, where skin pigmentation differs from the 'average' of their ancestral group. Therefore, use of a more objective classification method is essential to achieve accurate and reliable results.[6] Moreover, it is unclear to what extent older research remains relevant to current practice, given the refinements in pulse oximeters that have been made since those studies were conducted.

Current regulations and limitations

Pulse oximeters can be categorised into two general groups: those that are approved by the United States Food and Drug Administration (FDA) and available with a prescription and those that are not FDA-approved but widely available for purchase.[21] Medical oximeters can only be purchased through medical equipment distributors or by prescription. These oximeters are reviewed by the FDA and receive 510(k) authorisation. The FDA requires that these types of pulse oximeters undergo clinical testing to confirm accuracy. They are most commonly used in hospitals and physician's offices and are rarely used in the home.[21]

Oximeters for personal use can be purchased from technology stores or online, often accompanied by smartphone applications to provide real-time information on haemoglobin oxygen saturation. The demand for these oximeters has risen during the COVID-19 pandemic. Whilst these devices are typically not FDA-approved and do not undergo FDA review, they can still offer helpful information, albeit with potentially lower accuracy and reliability compared with regulated medical-grade devices. When selecting a pulse oximeter, it is crucial to consider factors, such as regulatory approvals, accuracy, and reliability. In recent years, specific non-prescription pulse oximeters have obtained regulatory approvals from agencies, such as the FDA or the European CE marking, verifying their ability to measure SpO_2. However, validation of these devices should ideally encompass diverse populations, including individuals with various skin tones. The exclusion of diverse populations in validation studies represents a limitation that must be addressed to ensure accurate readings for all individuals, irrespective of their skin tone.[13]

The FDA has already warned about the limitations of pulse oximeters and their potential for inaccuracy in certain circumstances. Whilst the level of inaccuracy might be small and clinically insignificant in some cases, there is a risk of inaccurate measurements, particularly at low oxygen saturation levels, which could go undetected. Therefore, caution must be exercised when interpreting accuracy.[22] For instance, if an FDA-approved pulse oximeter displays a reading of 90%, the actual blood oxygen saturation typically falls within the range of 86–94%. Pulse oximeter accuracy is highest between saturations of 90% and 100%, moderate between 80% and 90%, and lowest below 80% saturation.[21]

Many clinicians have not received sufficient education and training on the limitations of pulse oximetry, which can lead to unawareness of various factors influencing the accuracy and interpretation of readings.[23] This training is crucial and should be included in undergraduate and graduate health science curricula, along with supplementary training programmes. During the COVID-19 pandemic, we learned that patient safety relies on the ability of healthcare personnel to interpret SpO_2 readings correctly when utilising pulse oximetry.[23] Table 1 presents some of the existing limitations and factors that can affect the accuracy and interpretation of pulse oximeter readings.[22–25]

Low perfusion states, such as hypotension, hypothermia, and vasoconstriction, can result in reduced blood flow and thus lead to less reliable readings as underestimations. Conversely, overestimations can occur in individuals with darker skin tones or patients with certain abnormal haemoglobin variants, or with elevated carboxyhaemoglobin or methaemoglobin as in heavy smokers. The accuracy of pulse oximeters can also be affected by high haemoglobin levels caused by conditions, such as chronic hypoxaemia, obstructive sleep apnoea, and polycythaemia. It remains unclear whether readings will be normal, low, or influenced by technological limitations.

Patients with high haemoglobin concentrations can have a normal SaO_2, indicating that the partial pressure of oxygen (PaO_2) is sufficiently high to saturate all available haemoglobin molecules. In such cases, SpO_2 readings obtained from pulse oximeters would likely be normal. However, there can be situations where high haemoglobin levels exceed the capabilities

Table 1 Factors affecting pulse oximetry accuracy and interpretation of readings.

SpO_2 underestimation	SpO_2 overestimation	Photoplethysmography defacement
Peripheral perfusion	Skin pigmentation	Motion artifacts
Cold skin temperature	Hot skin temperature	HR >150 beats min^{-1}
Skin thickness	Haemoglobinopathies	Light pollution
	Hb F	
Nail polish or tattoos	High glycosylated Hb	Electromagnetic interference
Hypoxaemia	High carbon monoxide	Probe placement
Anaemia		
Low perfusion states	High carboxyhaemoglobin	Irregular heart rhythms
High altitude or hypoxia training	High methaemoglobin	Cardiac arrhythmias
Nerve-blocking medications	Heavy smokers	
I.V. or I.D. dyes		

of pulse oximeters to measure oxygen saturation accurately. Furthermore, motion artifacts, HR >150 beats min^{-1}, excessive ambient light, incorrect or loose placement of the probe on the finger or earlobe, electromagnetic interference, or irregular heart rhythms can interfere with the signal processing of the pulse oximeter, resulting in inaccurate readings.

Pulse oximeters might not be as accurate in patients with darker skin tones and might not always detect low oxygen saturations in patients with certain respiratory conditions. Currently, there is no consensus on how to address this issue for individuals with dark skin. It is recognised that pulse oximeters pose significant accuracy challenges when reading oxygen saturations in this population. Neither the current FDA guidance nor the recognised ISO standard for pulse oximeters provides a specific methodology for assessing skin pigmentation. Manufacturers have used various scales, such as the Fitzpatrick, Munsen, and von Luschan scales, leading to inconsistent categorisation of skin pigmentation data.[22] Therefore, it is highly recommended to improve pulse oximeters by incorporating a greater number of wavelengths. Clear handling instructions and modifications to labelling should also be provided to communicate their limitations to medical professionals and home users.[22]

A comprehensive understanding of measurement errors that can impact oximetry accuracy is essential for clinical decision-making. The FDA encourages clinicians to be aware of potential limitations of pulse oximeters, particularly for patients with dark skin pigmentation. There have been calls for improved designs of pulse oximeters specifically for use in dark-skinned individuals and a better understanding of how skin pigmentation affects oximetry accuracy and interpretation. By incorporating monochromatic LEDs and additional wavelengths, pulse oximeters might be able to differentiate between oxygenated and deoxygenated haemoglobin more accurately, even in patients with dark skin pigmentation or low oxygen levels.

Declaration of interest

The authors declare that they have no conflicts of interest.

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Appendix A. Supplementary data

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Supplementary Material. Improving Pulse Oximetry Accuracy in Dark-Skinned Patients: Technical Aspects and Current Regulations.

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Theoretical model

Classical Lambert-Beer Law

Pulse oximetry relies on the principle that oxygenated hemoglobin and deoxygenated hemoglobin have different absorption characteristics at specific wavelengths. Various algorithms and mathematical formulas are used in pulse oximeters to determine oxygen saturation accurately. According to the Classical Lambert-Beer Law, light intensity decreases exponentially with the length of the trajectory.

$$I_t = I_0 e^{\epsilon(\lambda) C d} \tag{1}$$

where I_t is the intensity of transmitted (or received) light, I_0 is the incident intensity, $\epsilon(\lambda)$ is the extinction coefficient or absorptivity at wavelength λ of tissue layers and chromophores such as haemoglobin, melanin, water, etc., C is the concentration of the absorbing substance, and d is the path length of light through the medium. The fraction of light that passes through a sample is called light transmittance (T) defined by

$$T = \frac{I}{I_0} = e^{-\epsilon(\lambda) C d} \tag{2}$$

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From this parameter, the undispersed absorbance or optical density of the medium $A(\lambda)$ of this process can be expressed as:

$$A(\lambda) = -ln(T) = \epsilon(\lambda)Cd \tag{3}$$

The Beer-Lambert law can be extended to a situation where we encounter more than one absorbing medium. If we know the respective extinction coefficients and the absorbance of the light measured at different wavelengths, the mathematical representation of this system is based on the superposition of the corresponding individual processes. Therefore, the total absorbance $A(\lambda)$ of light in a medium consisting of n absorbing substances will be the sum of their independent absorbances.

$$A(\lambda) = \epsilon_1(\lambda)C_1d_1 + \epsilon_2(\lambda)C_2d_2 + ... + \epsilon_n(\lambda)C_nd_n = \sum_{i=1}^{n} \epsilon_i(\lambda)C_id_i \tag{4}$$

Ratio analysis

The pulsatile nature of blood flow allows for differentiation between the absorbed light during systolic and diastolic phases of the cardiac cycle. The time-dependent arterial pulsation of the transmission signal, also called AC component, is separated from the total transmission signal. The strength of the AC component is only about 1 to 2 % of the total transmission. DC component of the transmission signal is defined as the transmission without blood volume pulsation. During one pulse, the time-derivative of the total absorbance or the differential absorption can be approximated with the $DC(\lambda)$ component and the $AC(\lambda)$ component intensities as:

$$\begin{aligned} dA(\lambda) &= \frac{dA(\lambda)}{dt}\Delta t = \frac{d_i}{dt}\sum_{i=1}^{n} \epsilon_i(\lambda)C_i\Delta t = \\ &= \frac{d(-ln(I_t(t)/I_0))}{dt}\Delta t = \frac{1}{I_0}\frac{I_t'(t)}{I(t)}\Delta t \simeq \frac{AC(\lambda)}{DC(\lambda)} = \frac{I_{max} - I_{min}}{I_{max}} \end{aligned} \tag{5}$$

where I_{min} is the minimum transmission after systolic rise, and I_{max} is diastolic maximum transmission of light. The ratio of two differential absorptions with different wavelengths, ratio-of-ratios (R) is used to calculate the oxygen saturation.

$$R = \frac{dA(\lambda_1)}{dA(\lambda_2)} = \frac{(\epsilon_{HbO_2}(\lambda_1) * C_{HbO_2} + \epsilon_{HHb}(\lambda_1) * C_{HHb})\Delta d(\lambda_1)}{(\epsilon_{HbO_2}(\lambda_2) * C_{HbO_2} + \epsilon_{HHb}(\lambda_2) * C_{HHb})\Delta d(\lambda_2)} \tag{6}$$

The extinction coefficients can be expressed in terms of blood saturation as [1]:

$$\epsilon = \epsilon_{HbO_2} SpO_2 + \epsilon_{HHb}(1 - SpO_2) \tag{7}$$

substituting Eq.7 in Eq.6 and assuming that $\Delta d(\lambda_1)/\Delta d(\lambda_2)$ remains constant, the oxygen saturation SpO_2 can be written as:

$$SpO_2 = \frac{\epsilon_{HHb}(\lambda_1) - R \cdot \epsilon_{HHb}(\lambda_1)}{\epsilon_{HHb}(\lambda_1) - \epsilon_{HbO_2}(\lambda_2) - R \cdot (\epsilon_{HHb}(\lambda_1) - \epsilon_{HbO_2}(\lambda_2))} \tag{8}$$

Although this straightforward derivation of pulse oximetry successfully establishes the mathematical foundation, it leaves out a crucial element: light scatters as it passes through living tissues. It has been shown that if at least two emitters of specific wavelengths are used, it is possible to obtain oxygen saturation by applying the principles of pulse oximetry. From the constant part *DC* and the variable part *AC* of the detected PPG signals for two wavelengths λ_1 and λ_2, a quotient of quotients can be obtained:

$$Q = \frac{AC(\lambda_1)/DC(\lambda_1)}{AC(\lambda_2)/DC(\lambda_2)} \tag{9}$$

which is related to oxygen saturation through:

$$SpO_2 = \frac{K_1 - K_2 Q}{K_3 - K_4 Q} \tag{10}$$

Where (K_1,K_2,K_3,K_4) are calibration coefficients for a real medium, which are related to the hemoglobin absorption coefficients ($\epsilon_{HHb}(\lambda_1)$, $\epsilon_{HbO_2}(\lambda_1)$, $\epsilon_{HHb}(\lambda_2)$,$\epsilon_{HbO_2}(\lambda_2)$) [2].

Empirical calibration curves

Oxygen saturation levels can also be obtained through empirical calibration curves. These curves are created by measuring the *R* ratio at various known oxygen saturation levels and fitting a mathematical function to the collected data. The specific form of the calibration curve may differ among different pulse oximetry devices, but a common generalization is as follows
[3, 4]:

$$SpO_2 = 110 - 25R \tag{11}$$

Although *R* represents the ratio between the red and infrared light signals, in practice, *R* can be obtained from the PPG signal using different methods [5].

Multi-wavelength method

A multi-wavelength system can be modeled by the Beer-Lambert law as presented before. Equation 5 can be represented in matrix notation as follows for a system with *n* wavelengths and *m* analytes [6].

$$\begin{bmatrix} dA(\lambda_1) \\ \vdots \\ dA(\lambda_n) \end{bmatrix} = \begin{bmatrix} \Delta d_{\lambda_1} \cdots 0 \\ \vdots \ddots \vdots \\ 0 \cdots \Delta d_{\lambda_n} \end{bmatrix} \cdot \begin{bmatrix} \epsilon_{\lambda_1}, HbX_1 \cdots \epsilon_{\lambda_1}, HbX_m \\ \vdots \ddots \vdots \\ \epsilon_{\lambda_n}, HbX_1 \cdots \epsilon_{\lambda_n}, HbX_m \end{bmatrix} \cdot \begin{bmatrix} HbX_1 \\ \vdots \\ HbX_m \end{bmatrix} \cdot C(Hb)$$

where $dA(\lambda)$ is the differential absorption within the Beer-Lambert model, ϵ_λ,*HbX* is the millimolar extinction coefficient, *HbX* are hemoglobin fractions, Δd_λ is the optical path-length for wavelength λ and $C(Hb)$ is the hemoglobin concentration. The path-length in the Beer-Lambert model is unrelated to the wavelength. A further point to consider is that this linear equation can only be resolved if *m* is equal to or larger than *n*, i.e., four wavelengths are required to identify four hemoglobin types.

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